

Next-to-leading-order temperature corrections to correlators in QCD

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Abstract

Corrections of order T^4 to vector and axial current correlators in QCD at a finite temperature $T < T_c$ are obtained using dispersion relations for the amplitudes of deep inelastic scattering on pions. Their relation with the operator product expansion is presented. An interpretation of the results in terms of T -dependent meson masses is given: masses of ρ and a_1 start to move with temperature in order T^4 .

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In the last years there has been an increasing interest in the study of the current correlators in QCD at finite temperatures. The hope is that investigating the same correlators, both at high temperatures, where the state of quark-gluon plasma is expected, and at low temperatures, where the hadronic phase persists, a clear signal for a phase transition could be found. For a review of calculations of correlators performed by various analytical methods and in the lattice simulations, see, e.g., Ref. [1].

The study of temperature dependence of current correlators is interesting in many aspects. At small distances the correlators are expressed through the operator product expansion (OPE) in terms of matrix elements of the operators of low dimension. In this way, the temperature dependence of these matrix elements manifests itself in the temperature dependence of the correlators and visa versa. At $T = 0$ using dispersion relations the correlators can be expressed in terms of contributions of hadronic states. Then, using some theoretical tools (differentiation, Borel transformation, etc) it is possible to enhance the contribution of lowest hadronic states. Therefore, knowledge of T -dependence of hadronic correlators can give us an information about the T -dependence of masses of the lowest hadronic states. Since in the approach of QCD sum rules, these masses are determined by the matrix elements of operators in OPE, the T -dependences of both are interrelated.

As argued in Ref. [2], in carrying out this program certain wrong steps had been taken, and a misunderstanding existed in the literature. Thus, some authors maintained that the thermal shift of hadron masses occurs already in the lowest order in temperature, $O(T^2)$. However, the general statement is that hadron masses do not move in this order [2, 3]. This statement is very general: it is based only on PCAC and current algebra. (The result that the nucleon pole does not move in order T^2 was obtained in the chiral perturbation theory in Ref. [4] and by considering a current correlator in Ref. [5]). The only interesting physical phenomenon that occurs in this order is the parity mixing, i.e. the appearance of states with opposite parity in a given channel and, in some cases, also an isospin mixing. The latter arises in baryon current correlators where, for example, in the current with the quantum numbers of Λ there appears a Σ pole, and in the nucleon channel there appear poles corresponding to baryon resonances with $J^P = \frac{1}{2}^\pm$ and $T = \frac{3}{2}, \frac{1}{2}$. For the case of vector and axial currents in the chiral limit, this mixing is given by

$$\begin{aligned} C_{\mu\nu}^V(q, T) &= (1 - \epsilon)C_{\mu\nu}^V(q, 0) + \epsilon C_{\mu\nu}^A(q, 0) \\ C_{\mu\nu}^A(q, T) &= (1 - \epsilon)C_{\mu\nu}^A(q, 0) + \epsilon C_{\mu\nu}^V(q, 0) \end{aligned} \quad (1)$$

where $C_{\mu\nu}^{V(A)}(q, T)$ are the correlators of V and A currents at finite temperature, $C_{\mu\nu}^{V(A)}(q, 0)$

are the same correlators at $T = 0$,

$$\epsilon = \frac{T^2}{6F_\pi^2}, \quad (2)$$

and $F_\pi = 93$ MeV is the pion decay constant.

At $T \neq 0$ both $C_{\mu\nu}^V$ and $C_{\mu\nu}^A$ have transverse and longitudinal parts

$$C_{\mu\nu}^{V(A)}(q, T) = (-g_{\mu\nu}q^2 + q_\mu q_\nu)C_t^{V(A)}(q^2, T) + q_\mu q_\nu C_l^{V(A)}(q^2, T) \quad (3)$$

At $T = 0$ $C_l^V(q^2, 0) = 0$, but $C_l^A(q^2, 0)$ is non-zero and in the chiral limit (when light quarks and pions are massless) is given by the one-pion contribution

$$C_l^A(q^2, 0) = \frac{F_\pi^2}{q^2} \quad (4)$$

According to Eq. (1), at $T \neq 0$ the longitudinal part (the pion pole) appears also in the vector channel.

If $C^{V,A}(q, 0)$ are represented through dispersion relations by contributions of the physical states in the V and A channels (ρ , a_1), then according to Eq. (1) the poles that are in the r.h.s. of Eq. (1), i.e. at $T = 0$, appear at the same positions in the l.h.s. Therefore, in order T^2 the poles corresponding to ρ and a_1 do not move [2]. An important consequence of Eq. (1) is that at $T \neq 0$ in the vector (transverse) channel apart from the poles corresponding to vector particles, there arise poles corresponding to axial particles and vice versa, i.e. a sort of parity mixing phenomenon occurs. The manifestation of this phenomenon is in complete accord with the general considerations presented in Ref. [2]: the appearance of the ρ (a_1) pole in the axial (vector) channel corresponds to singularities in the s -channel. In the same way a pion pole appears in the longitudinal part of the vector channel.

In this paper we show that in the next order, $O(T^4)$, such a simple picture, where the current correlator at finite temperature is represented by the superposition of $T = 0$ correlators, does not take place. Interpreted in terms of temperature-dependent poles, it would mean that masses are shifted in this order. In what follows we consider only the transverse part of the correlator because the T -dependence of the pion mass and the decay constant F_π was thoroughly investigated earlier [6], and we can say nothing new here.

The thermal correlation functions in Eq. (1) are defined as

$$C_{\mu\nu}(q, T) = \langle i \int d^4x e^{iqx} T \{ j_\mu(x), j_\nu(0) \} \rangle_T = \frac{\sum_n \langle n | i \int d^4x e^{iqx} T \{ j_\mu(x), j_\nu(0) \} e^{-H/T} | n \rangle}{\sum_n \langle n | e^{-H/T} | n \rangle}, \quad (5)$$

where H is the QCD Hamiltonian, the sum is over all states of the spectrum, and $j_\mu(x)$ is either a vector or an axial current, $j_\mu(x) = (1/2)(\bar{u}\gamma_\mu(\gamma_5)u - \bar{d}\gamma_\mu(\gamma_5)d)$. It is assumed that q^2 is space-like, $Q^2 = -q^2 > 0$, and Q^2 is much larger than a characteristic hadronic scale, $Q^2 \gg R_c^{-2}$, where R_c is the confinement radius, $R_c^{-1} \sim 0.5$ GeV.

We consider the case of temperatures T below the phase transition temperature T_c . In principle, the summation over n in Eq. (5) can be performed over any complete set of states $|n\rangle$ in the Hilbert space. It is clear however, that at $T < T_c$ the suitable set of states is the set of hadronic states, but not the quark-gluon basis. Indeed, in this case the original particles forming the heat bath, which is probed by external currents, are hadrons. The summation over the quark-gluon basis of states would require to take into account the full range of their interaction. In connection with consideration of current correlators at finite T , this point was first explicitly made in Ref. [3]. In the early papers [7] devoted to the extension of QCD sum rules to finite temperatures the summation over $|n\rangle$ at low T was performed in the quark-gluon basis without account of confinement. In a recent paper [8] QCD sum rules at finite T were rewritten using the pion basis.

At T well below the phase transition temperature T_c an expansion in T can be performed. The main contribution comes from the pion states, $|n\rangle = |\pi\rangle, |2\pi\rangle, \dots$. In this paper we restrict ourselves to the chiral limit, when u, d quarks and pions are massless. The corrections to the chiral limit will be considered in a separate publication.

In the chiral limit there are two parameters in the low T expansion. One parameter appears when the pion momenta, $p \sim T$, in the matrix elements in Eq. (5) can be neglected. Then, the matrix elements in Eq. (5) can be calculated using PCAC and current algebra. The powers of T^2 arise due to phase space integration with the Bose factor. In this case the expansion parameter is T^2/F_π^2 : the one-pion contribution is proportional to T^2/F_π^2 , the two-pion contribution is of order T^4/F_π^4 , etc [3]. In the order T^2 there are only terms of this type. It is clear that in any order $(T^2)^n$ the terms of this type are expressed through the vector and axial vacuum correlators $C_{\mu\nu}^{V(A)}(q^2, 0)$ and, as a consequence, do not result in thermal shifts of hadron masses. The other expansion parameter appears if non-vanishing pion momenta in the matrix elements in Eq. (5) are taken into account. Since the characteristic distances in Eq. (5) are of order $x^2 \sim 1/Q^2$, the expansion parameter in this case is T^2/Q^2 . It is also worth to mention that the contributions of massive hadronic states $|n\rangle$ are exponentially suppressed as $\exp(-m_n/T)$.

Let us start with the calculation of T^4 terms of the first type. The matrix elements over two-pion states in the limit $p \rightarrow 0$ give a T^4 contribution. But this is not just a second iteration of the procedure used to obtain the one-pion matrix element. It is also

necessary to take into account the interaction between the pions in the initial and the finite states. This can be illustrated by the example of T^4 terms in the T -dependence of the quark condensate[6]:

$$\langle \bar{q}q \rangle_T = \langle \bar{q}q \rangle_0 \left(1 - \frac{3}{4}\epsilon - \frac{3}{32}\epsilon^2 \right) \quad (6)$$

The ϵ and ϵ^2 terms here come from one- and two-pion matrix elements, respectively. However, not accounting for the initial (finite) state interaction of pions in the two-pion state would give $9\epsilon^2/32$ instead. The interaction amplitude for $\pi^b \pi^d \rightarrow \pi^a \pi^c$ to zeroth order in pion momenta is given by

$$\frac{m_\pi^2}{F_\pi^2} (3\delta_{ac}\delta_{bd} - \delta_{ab}\delta_{cd} - \delta_{ad}\delta_{bc}). \quad (7)$$

The m_π^2 in the numerator of the above amplitude cancels against the pion mass in the pion propagator at $p \rightarrow 0$ and contributes $-3\epsilon^2/8$ to the T -dependence of $\langle \bar{q}q \rangle$, making the correct total of $-3\epsilon^2/32$.

Similarly, it is easy to show that in the case of vector and axial correlators the two-pion matrix elements with the account of initial (finite) state interaction of pions result in the following expressions for the transverse part of V and A correlators:

$$C_{\mu\nu}^{V(A)}(q, T) = (-g_{\mu\nu}q^2 + q_\mu q_\nu) \left(C_{V(A)}(q^2) + \left(\epsilon - \frac{1}{2}\epsilon^2 \right) (C_{A(V)}(q^2) - C_{V(A)}(q^2)) \right). \quad (8)$$

As was expected, the T^4/F_π^4 terms above are again expressed through correlators at $T = 0$, and they do not result in a thermal mass shift. Also, these terms are Lorentz invariant, since only the tensor $-g_{\mu\nu}q^2 + q_\mu q_\nu$ appears here.

Consider now the terms of the second type, that arise from non-zero pion momenta. It is well known from the description of deep inelastic electron-hadron scattering¹ that a matrix element of the product of two vector currents may be represented using two tensor structures

$$\begin{aligned} T_{\mu\nu}^\pi(p, q) &= i \int d^4x e^{iqx} \langle \pi(p) | T \{ j_\mu(x), j_\nu(0) \} | \pi(p) \rangle \\ &= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) T_1(\nu, q^2) + \left(p_\mu - \nu \frac{q_\mu}{q^2} \right) \left(p_\nu - \nu \frac{q_\nu}{q^2} \right) T_2(\nu, q^2), \end{aligned} \quad (9)$$

¹We are now considering the case of a vector correlator. However, the axial correlator in the chiral limit has the same tensor structure, and the results obtained in the vector case may be directly applied to the axial one.

where $\nu = pq$, T_1 is dimensionless, and T_2 has dimension $mass^{-2}$. The corresponding contribution to the thermal correlation function is obtained by integrating the above equation over the pion phase space with the Bose factor. And it is the second term in the r.h.s. of Eq. (9) that, after this integration, provides the expected Lorentz non-invariant contribution to the thermal correlator. In terms of the OPE, the function T_2 is contributed only by averages of Lorentz non-scalar operators, while T_1 receives contributions from both Lorentz scalar and non-scalar operators[9].

It is well known[10] that the function T_2 satisfies a dispersion relation without subtractions:

$$T_2(\nu, q^2) = \frac{2}{\pi} \int_{Q^2/2}^{\infty} \frac{\nu' \text{Im } T_2(\nu', q^2) d\nu'}{\nu'^2 - \nu^2}. \quad (10)$$

Having in mind the subsequent integration over p , we are interested in $T_2(0, q^2)$. Then, using the relation between the imaginary part of T_2 and the structure function F_2

$$\text{Im } T_2(\nu, q^2) = \frac{2\pi}{\nu} F_2(x, q^2), \quad (11)$$

where $x = Q^2/2\nu$ is the standard deep inelastic scaling variable, we get

$$T_2(0, q^2) = \frac{8}{Q^2} \int_0^1 F_2(x, q^2) dx. \quad (12)$$

Similarly, the function $T_1(\nu, q^2)$ satisfies a dispersion relation with one subtraction,

$$T_1(\nu, q^2) - T_1(0, q^2) = \frac{2\nu^2}{\pi} \int_{Q^2/2}^{\infty} \frac{\text{Im } T_1(\nu', q^2) d\nu'}{\nu'(\nu'^2 - \nu^2)}. \quad (13)$$

Again, using the relation between $\text{Im } T_1$ and the structure function F_1

$$\text{Im } T_1(x, q^2) = 2\pi F_1(x, q^2), \quad (14)$$

we obtain

$$T_1(\nu, q^2) - T_1(0, q^2) = \frac{8\nu^2}{Q^4} \int_0^1 2xF_1(x, q^2) dx. \quad (15)$$

The subtraction constant $T_1(0, q^2)$ corresponds to zero momenta of the initial and final pions and was already accounted for in the terms proportional to T^2/F_π^2 . It is worth to mention that at this stage the assumption $Q^2 \gg R_c^2$ was not used. In the derivation of Eqs. (12) and (15) it was assumed that $\nu \ll Q^2$, which is equivalent in the chiral limit to $Q > T$. The same assumption was sufficient for the derivation of Eq. (8). Therefore, Eqs. (8), (12) and (15) are correct also at moderate $Q^2 \sim 1 \text{ GeV}^2$.

At higher Q^2 , in the scaling region, the integrals in Eqs. (12) and (15) are equal, since in this region $2xF_1(x, Q^2) = F_2(x, Q^2)$. The integral

$$M_2 = \int_0^1 F_2(x) dx \quad (16)$$

is the second moment of the structure function and in the parton model has the meaning of the fraction of the pion momentum carried by quarks. In our normalization of currents

$$M_2 = \frac{1}{4} \cdot \frac{1}{3} \int_0^1 x dx \sum_{q,a} [q_{\pi^a}(x) + \bar{q}_{\pi^a}(x)] = \frac{1}{2} \int_0^1 x dx [v(x) + 2s(x)], \quad (17)$$

where $v(x)$ and $s(x)$ are the distributions of valence and sea quarks in the pion (see, e.g. [10]). The factor $1/4$ in Eq. (17) comes from the definition of the currents in Eq. (5), and the factor $1/3$ arises due to averaging over π^+ , π^- , π^0 in the heat bath.

The distributions $v(x)$ and $s(x)$ were parametrized in Ref. [12] to fit the experimental data on the Drell-Yan process $\pi + N \rightarrow l^+ l^- + X$ and on the direct photon production $\pi + N \rightarrow \gamma + X$, and it was found that $M_2 \approx 0.12$, which is somewhat lower than the result 0.15 ± 0.02 of Ref. [11], where this quantity was obtained from QCD sum rules for a correlation function in external symmetric tensor field, and close to 0.11 ± 0.03 as estimated in Ref. [9]. These numbers correspond to the normalization point $\mu = 1$ GeV.

Now, to obtain the corresponding contribution to the thermal correlator, one has just to do the Bose-weighted integrals over the pion momentum and sum over the three pions

$$3 \int \frac{d^3 p}{(2\pi)^3 2p} \frac{1}{\exp(|up|/T) - 1} T_{\mu\nu}^\pi(p, q) \quad (18)$$

(u is the 4-velocity of the heat bath), which gives together with Eq. (8)

$$C_{\mu\nu}(q, T) = \left(-g_{\mu\nu} q^2 + q_\mu q_\nu\right) C_1(T, q^2) + u_\mu^t u_\nu^t C_2(T, q^2), \quad (19)$$

where $u_\mu^t = u_\mu - (uq)q_\mu/q^2$,

$$\begin{aligned} C_1(T, q^2) &= C_V(0, q^2) + \left(\epsilon - \frac{1}{2}\epsilon^2\right) \left(C_A(q^2) - C_V(q^2)\right) + \frac{1}{2} c \frac{1 + 2\mathbf{q}^2/q^2}{q^4} T^4 \\ C_2(T, q^2) &= -c \frac{T^4}{q^2}, \end{aligned} \quad (20)$$

and

$$c = \frac{8\pi^2 M_2}{15}. \quad (21)$$

The low temperature expansion of C_1 contains powers of both T^2/F_π^2 and T^2/Q^2 , but powers of T^2/F_π^2 are absent in C_2 . Notice, that while all three pion charge states contribute to T^2/Q^2 terms, only two of them contribute to T^2/F_π^2 terms. The same formulae hold for the axial correlator, with the obvious change $V \leftrightarrow A$.

The above formulae for the thermal correlator may be considered also from the viewpoint of the OPE for the correlation function (see, Refs. [8, 9]). The OPE itself of course carries no information hey about the state over which the matrix element of the operators is considered, or about the heat bath, in case of finite T . It contains operators of arbitrary Lorentz spin s and twist t . When vacuum correlators are considered, only scalar, $s = 0$, operators contribute, while non-scalar, $s \neq 0$, operators drop out in the averaging. When averaging over a hadron state or over a heat bath, the $s \neq 0$ operators do contribute, since there is an additional vector in the problem, the target momentum p or the 4-velocity of the heat bath u . The matrix elements of $s = 0$ operators over pions or the heat bath may be estimated (if the pion momenta can be neglected) using PCAC, which relates them to vacuum averages. The $s = 0$ operators do not give $(T^2/Q^2)^n$ terms and therefore do not result in thermal mass shifts. The general expression for the hadron matrix element of an $s = n$, $n \geq 2$ operator is $\langle p | \hat{O}_{\mu_1, \mu_2, \dots, \mu_n} | p \rangle = a p_{\mu_1} p_{\mu_2} \dots p_{\mu_n}$ (in the chiral limit there are no trace terms $\sim g_{\mu_i \mu_j}$). These matrix elements cannot be reduced by PCAC to vacuum averages and are new non-perturbative parameters. Their Bose-weighted integrals over the pion momenta are T -dependent $s \neq 0$ condensates which are suppressed as T^s compared to the T -dependent parts of $s = 0$ condensates.

It is clear that in terms of the OPE the function C_1 is contributed both by $s = 0$ and $s \neq 0$ condensates, as is T_1 . However, C_2 and T_2 are related only to the $s \neq 0$ condensates.

In the chiral limit a difference in the $s = 0$ operators for the vector and axial correlators appears on the level of 4-quark operators². A good consistency check of the calculation of correlators in the pion gas approximation is on whether the T dependences of the $s = 0$ 4-quark condensates in $C_1^{V(A)}$ match the $V - A$ mixing in the first of Eqs. (20). This indeed turned out to be the case, as demonstrated (to order T^2) in Ref. [14], and also in Ref. [15] for baryonic currents. The correlation in T -dependences of the $s = 0$ condensates in opposite parity channels is not accidental and is related to the *scattering* of thermal pions on the currents. Notice also, that since

$$C_1^V(T) - C_1^A(T) = (1 - 2\epsilon + \epsilon^2) (C_1^V(0) - C_1^A(0)) \quad (22)$$

²The gluon condensate gets its T dependence only in order T^8 [13], anyway.

this correlation exactly satisfies Weinberg sum rules generalized [16] to finite T .

Among the non-singlet condensates, the leading contribution to C_2 at low T comes from the lowest spin, $s = 2$, which corresponds to the T^4 behavior. In the leading twist there are two $s = 2$ operators which are related to the energy-momentum tensors of quarks, $\theta_{\lambda\sigma}^q$, and gluons, $\theta_{\lambda\sigma}^G$,

$$\begin{aligned}\theta_{\lambda\sigma}^q &= \frac{i}{2}(\bar{q}\gamma_\lambda D_\sigma q + \bar{q}\gamma_\sigma D_\lambda q), \quad q = u, d \\ \theta_{\lambda\sigma}^G &= G_{\lambda\alpha}^a G_{\alpha\sigma}^a - \frac{1}{4}g_{\lambda\sigma} G_{\alpha\beta}^a G_{\beta\alpha}^a.\end{aligned}\quad (23)$$

Explicit expressions for the contribution of these operators to T_1 and T_2 can be obtained from the general formulae of the theory of deep inelastic scattering (see, e.g. [17]). We present here the result for the case, when the longitudinal structure function $F_L = 2xF_1(x) - F_2(x)$ is neglected and only QCD corrections proportional to $\alpha_s \ln(Q^2/\mu^2)$ are retained. It can be shown that since all pion charge states are equally populated in the heat bath, only flavour-singlet operators contribute to the structure functions. Then

$$\begin{aligned}T_1(\nu, q^2) &= \frac{\nu}{2x} T_2(\nu, q^2) \\ &= \frac{q_\lambda q_\sigma}{q^4} \left[\left(1 - \frac{8}{9} \frac{\alpha_s}{\pi} \ln(Q^2/\mu^2) \right) \langle \pi | \Sigma_q \theta_{\lambda\sigma}^q | \pi \rangle + \frac{\alpha_s}{8\pi} \ln(Q^2/\mu^2) \langle \pi | \theta_{\lambda\sigma}^G | \pi \rangle \right],\end{aligned}\quad (24)$$

where the averaging over the three pion charge states must be performed. It is easy to see that the contribution of $\theta_{\lambda\sigma}^G$ to Eq. (24) is small at $Q^2 \sim 1 \text{ GeV}^2$.

The pion matrix element of the total energy-momentum tensor is just a normalization constant,

$$\langle \pi(p) | \theta_{\mu\nu}^u + \theta_{\mu\nu}^d + \theta_{\mu\nu}^G | \pi(p) \rangle = 2p_\mu p_\nu \quad (25)$$

($\langle \pi(p) | \pi(p') \rangle = (2\pi)^3 2E \delta^{(3)}(\mathbf{p} - \mathbf{p}')$). On the other hand, we have

$$\langle \pi(p) | \theta_{\mu\nu}^u + \theta_{\mu\nu}^d | \pi(p) \rangle = 8M_2 p_\mu p_\nu. \quad (26)$$

So, if we define also a constant b as $\langle \pi | \theta_{\mu\nu}^G | \pi \rangle = b p_\mu p_\nu$, then $8M_2 + b = 2$. The constant b enters the matrix element $\langle \pi | \mathbf{E}^2 + \mathbf{B}^2 | \pi \rangle$ and also $\langle \mathbf{E}^2 + \mathbf{B}^2 \rangle_T$. It was determined in Ref. [9] from a duality type of consideration: $b = 1.16 \pm 0.14$ at $\mu = 1 \text{ GeV}$. This is in accord with the estimates for M_2 obtained in Refs. [8, 11] and with the statement that gluons carry about 50% of nucleons.

Now, we would like to discuss a possibility to interpret the T^4/Q^2 corrections to the correlators in Eqs. (20) in terms of particle thermal mass shifts. It is convenient to use the standard representation [7] of the vector correlator in a medium in terms of two invariant functions C_l and C_t , which in the rest frame of the heat bath ($u = (1, \mathbf{0})$) are defined as

$$\begin{aligned} C_{00}^T &= \mathbf{q}^2 C_l^T \\ C_{ij}^T &= \left(\delta_{ij} - \frac{q_i q_j}{\mathbf{q}^2} \right) C_t^T + \frac{q_i q_j}{\mathbf{q}^2} q_0^2 C_l^T, \end{aligned} \quad (27)$$

or in terms of C_1 and C_2

$$\begin{aligned} C_t &= q^2 C_1 \\ C_l &= C_1 + \frac{\mathbf{q}^2}{q^4} C_2. \end{aligned} \quad (28)$$

At $T = 0$ these two functions are not independent, $C_l = C_t/q^2 = C_V$. They are also related at $T \neq 0$, if

$$\underline{\mathbf{q} = 0, q_0 \neq 0, Q^2 = -q_0^2}$$

$$\begin{aligned} C_1^V(T) - C_1^V(T=0) &= C_l^T - C_l^{T=0} = -\frac{1}{Q^2} (C_t^T - C_t^{T=0}) \\ &= -\left(\epsilon - \frac{1}{2}\epsilon^2 \right) (C_1^V - C_1^A) + c \frac{T^4}{2Q^2}, \end{aligned} \quad (29)$$

where c was defined in Eq. (21).

In another special case

$$\underline{q_0 = 0, \mathbf{q} \neq 0, Q^2 = \mathbf{q}^2}$$

$$\begin{aligned} Q^2(C_l^T - C_l^{T=0}) &= \left(\epsilon - \frac{1}{2}\epsilon^2 \right) (C_V - C_A) + c \frac{T^4}{2Q^2} \\ C_t^T - C_t^{T=0} &= -\left(\epsilon - \frac{1}{2}\epsilon^2 \right) (C_V - C_A) + c \frac{T^4}{2Q^2}, \end{aligned} \quad (30)$$

Till now we considered the correlators at negative $q^2 = -Q^2$. In order to interpret the results in terms of the particle mass shifts, the amplitudes at negative q^2 must be represented using dispersion relations through the contributions of physical states, defined at positive

q^2 and s . Unlike the case of $T = 0$, where the correlators are functions of one variable, q^2 , at $T \neq 0$ they are functions of two variables, q_0 and \mathbf{q}^2 . In this case the only way to represent the amplitude at negative q^2 through the contributions of physical states is to use the dispersion relation in q_0 at fixed \mathbf{q}^2 . (In the opposite case, when q_0 is fixed and \mathbf{q}^2 is variable, the amplitudes would have non-physical singularities).

So, let us consider the case $\mathbf{q} = 0$, $q_0 \neq 0$, $Q^2 = -q_0^2$. In this case there is only one structure function, $C_l(q_0) = C_l(q_0)$. We choose the standard model for the spectral densities as a sum of the lowest resonances and continuum. The dispersion relations over q_0 are contributed by the physical states in the q^2 - and s -channels. Therefore, for the structure function $C_1^V(Q^2, T)$ we have

$$C_1^V(Q^2, T) = \frac{\lambda_{\rho,V}^2(T)}{Q^2 + m_\rho^2(T)} + \frac{\lambda_{a_1,V}^2(T)}{Q^2 + m_{a_1}^2(T)} + \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\rho_V(s, T)}{Q^2 + s} ds, \quad (31)$$

where the first and the second terms in the r.h.s. of Eq. (31) correspond to the contributions of ρ and a_1 mesons to the vector current correlator, $\lambda_{\rho,V}^2(T)$ and $\lambda_{a_1,V}^2(T)$ are the corresponding T -dependent coupling constants, and $\lambda_{a_1,V} \sim T^2$. (The subtraction constant is omitted). A similar equation holds for the axial current correlator $C_1^A(Q^2, T)$. From Eq. (29) it is easy to see that the terms of order T^4/Q^2 vanish in the difference $C_1^V(Q^2, T) - C_1^A(Q^2, T)$:

$$\begin{aligned} C_1^V(Q^2, T) - C_1^A(Q^2, T) &= [C_1^V(Q^2, 0) - C_1^A(Q^2, 0)] \\ &= -\epsilon \left(1 - \frac{\epsilon}{2}\right) [C_1^V(Q^2, 0) - C_1^A(Q^2, 0)]. \end{aligned} \quad (32)$$

If we put

$$m_\rho^2(T) = m_\rho^2 + \delta m_\rho^2, \quad m_{a_1}^2(T) = m_{a_1}^2 + \delta m_{a_1}^2, \quad (33)$$

then from Eqs. (31) and (32) it follows that

$$\lambda_\rho^2 \delta m_\rho^2 - \lambda_{a_1}^2 \delta m_{a_1}^2 = 0 \quad (34)$$

and

$$\begin{aligned} \frac{\lambda_{\rho,V}^2(T)}{\lambda_\rho^2} &= \frac{\lambda_{a_1,A}^2(T)}{\lambda_{a_1}^2} = 1 - \epsilon \left(1 - \frac{1}{2}\epsilon\right) \\ \frac{\lambda_{\rho,A}^2(T)}{\lambda_\rho^2} &= \frac{\lambda_{a_1,V}^2(T)}{\lambda_{a_1}^2} = \epsilon \left(1 - \frac{1}{2}\epsilon\right). \end{aligned} \quad (35)$$

For the sum $C_1^V(Q^2, T) + C_1^A(Q^2, T)$ from Eq. (29) we have

$$C_1^V(Q^2, T) + C_1^A(Q^2, T) - [C_1^V(Q^2, 0) + C_1^A(Q^2, 0)] = c \frac{T^4}{Q^2}. \quad (36)$$

It is clear from the comparison of Eqs. (31) and (36) that with our model of hadronic spectrum the continuum cannot contribute to the l.h.s. of Eq. (36), since the imaginary part of the r.h.s. vanishes at $Q^2 < 0$. Then from Eqs. (31) and (36) we have

$$\lambda_\rho^2 \delta m_\rho^2 + \lambda_{a_1}^2 \delta m_{a_1}^2 = -c T^4. \quad (37)$$

(Taking into account only terms $\sim T^4$, we put $\lambda_\rho^2(T) \approx \lambda_\rho^2(0) = \lambda_\rho^2$ and $\lambda_{a_1}^2(T) \approx \lambda_{a_1}^2(0) = \lambda_{a_1}^2$). From Eqs. (34) and (37) we have

$$\delta m_\rho^2 = -c \frac{T^4}{2\lambda_\rho^2}, \quad \delta m_{a_1}^2 = -c \frac{T^4}{2\lambda_{a_1}^2}. \quad (38)$$

The residues in Eq. (38) are the standard couplings of ρ and a_1 mesons with the vector and axial currents, $\lambda_\rho^2 = m_\rho^2/g_\rho^2$, $\lambda_{a_1}^2 = m_{a_1}^2/g_{a_1}^2$. Numerically they are rather close, $\lambda_\rho^2 \approx \lambda_{a_1}^2 \approx 0.02$ [3].

We see that both the ρ and a_1 masses start decreasing with T , and the mass shifts appear in order T^4 and, in terms of OPE, are due to Lorentz non-scalar condensates as emphasized in refs.[9, 14]³. The corrections proportional to powers of T^2/F_π^2 affect only the residues of the currents. This fact can be easily understood. Indeed, in the OPE for the correlators taking into account finite temperatures to order T^2 would result only in the change of the same Lorentz scalar condensates which appear in OPE at $T = 0$. Then it is clear from the representation through dispersion relations that any such change can be described by modifications of the residues without affecting the position of poles.

The scenario when both the vector and axial masses decrease with T is allowed by Weinberg sum rules at $T \neq 0$ [16]. Numerically the mass shifts are rather small. Even at $T = 150 - 200$ MeV (usually accepted values for the phase transition temperature) $\delta m_\rho^2 \approx \delta m_{a_1}^2 \sim -(0.01 - 0.02) \text{ GeV}^2$. At the same time at these temperatures the change of residues according to Eq. (35) is very essential. (For this reason Eqs. (38) are not completely reliable at $T > 100$ MeV, since we put $\lambda^2(T) \approx \lambda^2(0)$).

³In Ref. [8] it was claimed that the ρ and a_1 masses move in order T^2 . This conclusion comes from an incorrect choice of the spectral density in the sum rules: the mixing of vector and axial channels was not taken into account. For this reason the results obtained in this paper for the T -dependence of the masses are not reliable, though we agree with the analysis of T -dependent condensates carried out there.

Let us summarize our main results. The corrections of order T^4 to the correlators of vector and axial currents were calculated in QCD in a model independent way in the chiral approximation when u , d quarks and pions are massless. The results are expressed in terms of the second moment of the pion structure function in deep inelastic lepton-pion scattering that is equal to the matrix element of the quark energy-momentum tensor over the pion state, or to the fraction of the pion momentum carried by quarks in the parton model. Interpreted in terms of physical mesons the calculated corrections correspond to negative mass shifts of the ρ and a_1 mesons proportional to T^4 . (As was shown earlier [2, 3], the mass shifts are absent in order T^2). These mass shift originate from Lorentz non-scalar condensates in OPE. Numerically they are rather small at $T \leq 100$ MeV, where our approach is correct. The corrections arising from a finite pion mass were not touched in this paper. We plan to consider them in a future work.

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